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Modeling Phase Transitions in Rapidly Expanding Systems

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HG with excluded volume correction

following Rischke, Gorenstein, Stöcker, Greiner, Z. Phys. C51 (1991) 485

$$P = \sum_i P_i^{id}(\mu_i - Pv_i, T)$$

Sum runs over all hadronic species

P_i^{id} – pressure of ideal gas

$v_i = v \sim (0.5—2)\ \mathrm{fm}^3$ excluded volume

Chemical potential for species i

$$\mu_i = \mu_B B_i + \mu_S S_i$$

Baryonic charge   Strangeness

μ_S is determined from the net strangeness neutrality

$$n_S = 0$$

$$n = (\partial_\mu P)_T = \frac{\sum_i n_i^{id}(\mu_i - Pv_i, T)}{1 + \sum_i n_i^{id}(\mu_i - Pv_i, T) v_i}$$

$$s = (\partial_T P)_\mu = \frac{\sum_i s_i^{id}(\mu_i - Pv_i, T)}{1 + \sum_i n_i^{id}(\mu_i - Pv_i, T) v_i}$$

$$\varepsilon = Ts + n\mu + n_S \mu_S - P \quad \Rightarrow \quad P = P(\varepsilon, n)$$

EOS suitable for hydro simulations

Hadronic species included

$$i = \begin{cases} M = \pi, \rho, \omega, \ldots, K, \overline{K}, \ldots (bosons) \rightarrow i \leq N_B = 59 \\ B = N, \Delta, \Lambda, \Sigma, \ldots (fermions) \rightarrow i \leq N_F = 41 \\ \overline{B} = \overline{N}, \overline{\Delta}, \overline{\Lambda}, \overline{\Sigma}, \ldots (fermions) \rightarrow i \leq N_F = 41 \end{cases}$$

We take into account all known hadrons with m ≤ 2 GeV, apart of $f_0(600)$, this set is very similar to THERMUS :

Wheaton & Cleymans, hep-ph/0407174

Calculations are done within zero-width approximation using Bose and Fermi statistics

$$P_M^{id} = \sum_{i \in M} \frac{g_i}{6\pi^2} \int_{m_i}^{\infty} dE \frac{(E^2 - m_i^2)^{3/2}}{e^{(E-\mu_i)/T} - 1} \Rightarrow T^2 \sum_{i \in M} \frac{g_i m_i^2}{2\pi^2} \sum_{n=1}^{\infty} \frac{K_2(m_i n / T)}{n^2}$$

$$P_B^{id} = \sum_{i \in B} \frac{g_i}{6\pi^2} \int_{m_i}^{\infty} dE \left[\frac{(E^2 - m_i^2)^{3/2}}{e^{(E-\mu_i)/T} + 1} + \frac{(E^2 - m_i^2)^{3/2}}{e^{(E+\mu_i)/T} + 1} \right]$$

Quark-Gluon phase within the Bag model

$$P_Q(\mu,T)=(\tilde{N}_g+\frac{21}{2}\tilde{N}_f)\frac{\pi^2}{90}T^4+\tilde{N}_f(\frac{T^2\mu^2}{18}+\frac{\mu^4}{324\pi^2})+$$
$$+\frac{1-\xi}{\pi^2}\int_{m_s}^{\infty}d\varepsilon(\varepsilon^2-m_s)^{3/2}\left\{\left[e^{\frac{\varepsilon-\mu_s}{T}}+1\right]^{-1}+\left[e^{\frac{\varepsilon+\mu_s}{T}}+1\right]^{-1}\right\}-B$$

$$\tilde{N}_g=16(1-0.8\xi)$$

$$N_f=2(1-\xi)$$

perturbative correction

$(\xi\sim\alpha_s)$

ξ, B, m_s – parameters of the model

$\xi=0.2$ extracted from lattice data

$m_s = 150$ MeV

$B^{1/4}=230$ MeV/fm^3

$\Downarrow$

$T_c(n=0)=165$ MeV

for u,d quarks

$$\mu_q=\frac{\mu}{3}$$

for s quarks

$$\mu_s=\frac{\mu}{3}-\mu_S$$

Phase transition HG-QGP

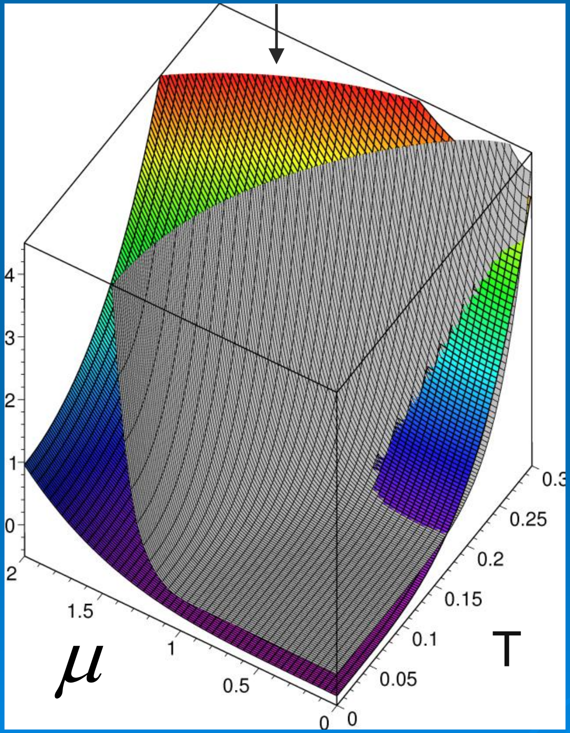
Gibbs criterion for phase transition: $\boxed{P_H(\mu_B,T) = P_Q(\mu_B,T)}$

Compare pressures of two phases as functions of T, μ

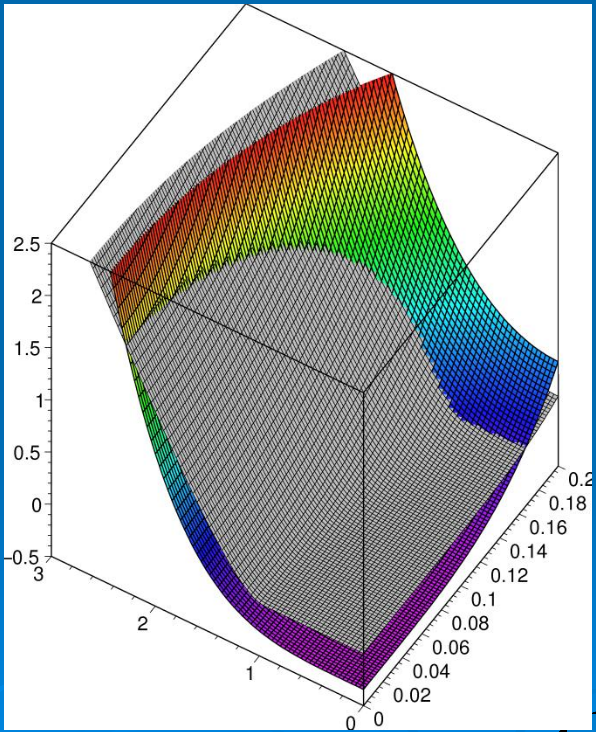
unphysical phase diagram

Hadronic phase

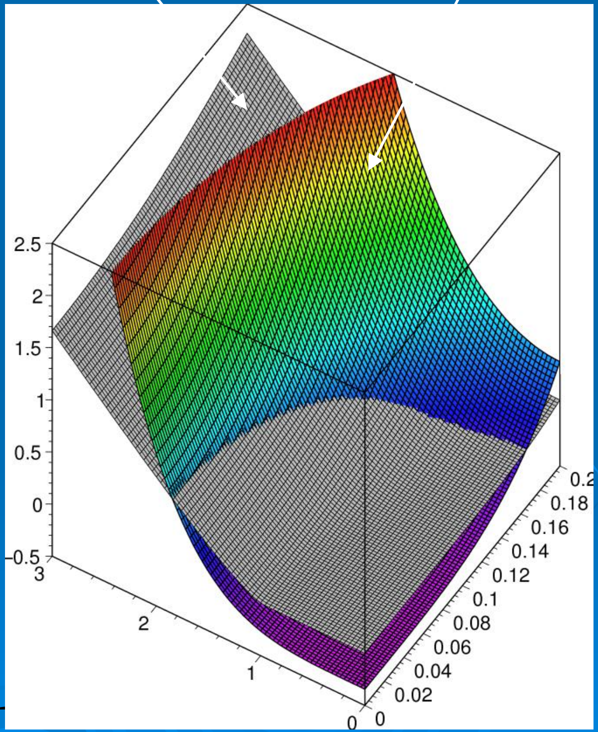
Quark phase



$v = 0$

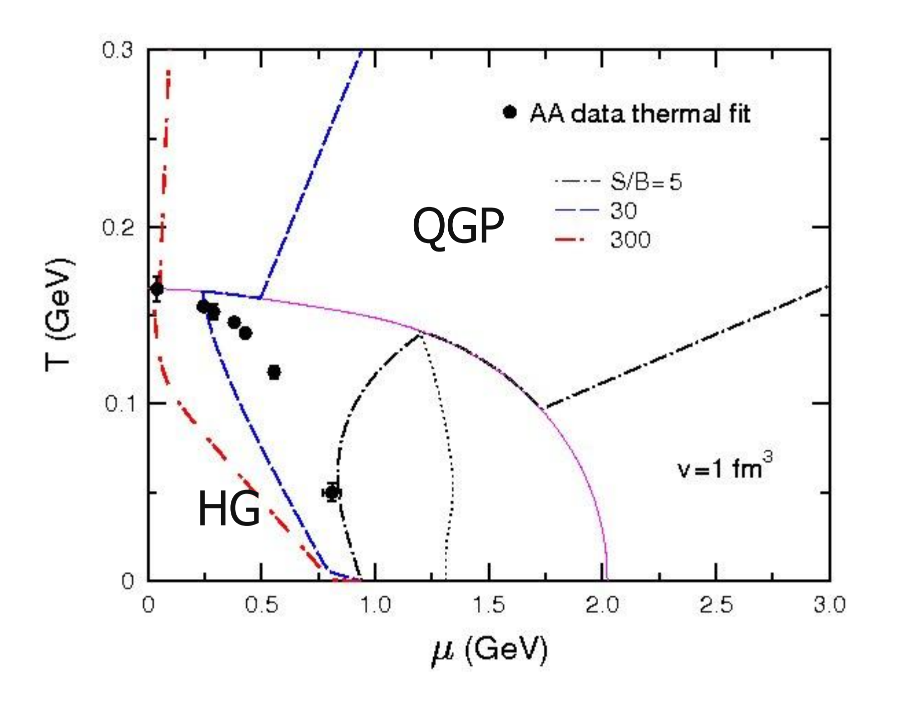


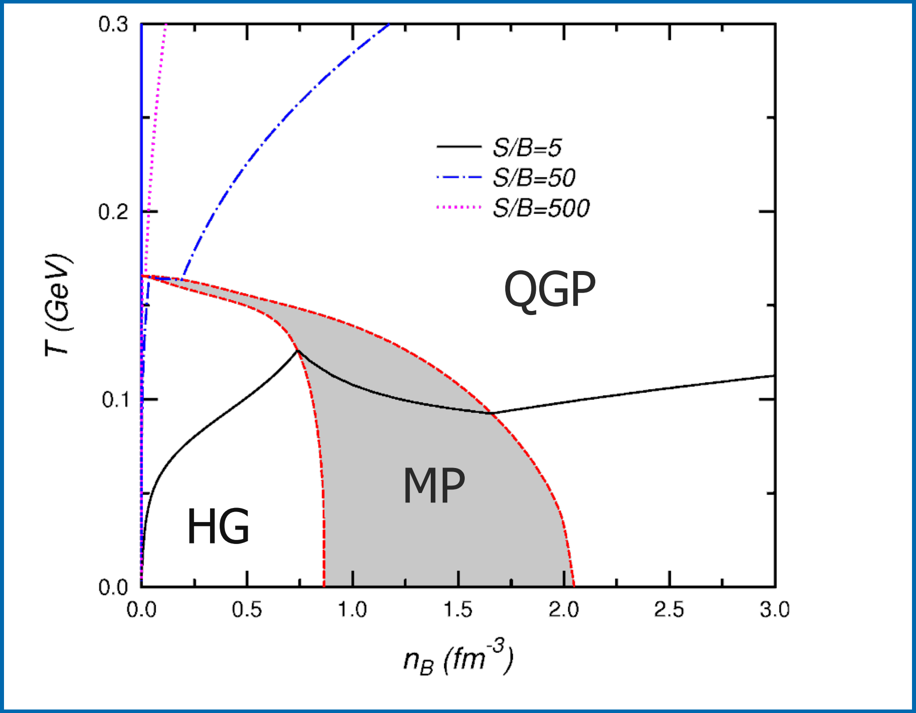
$v = 0.5\ \mathrm{fm}^3$



$v = 1\ \mathrm{fm}^3$

Adiabatic trajectories





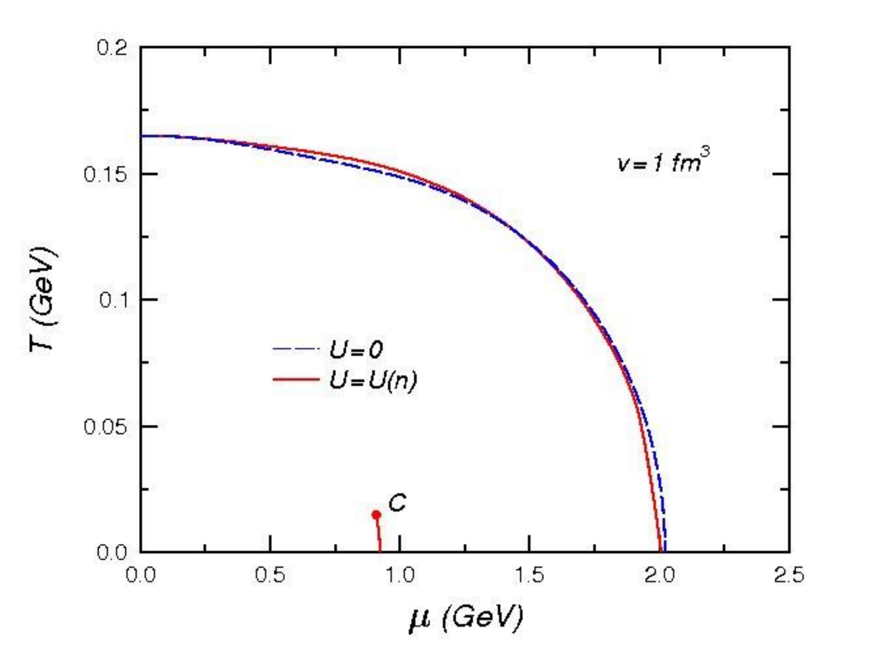
Temperature increases at transition from quarks to hadrons.

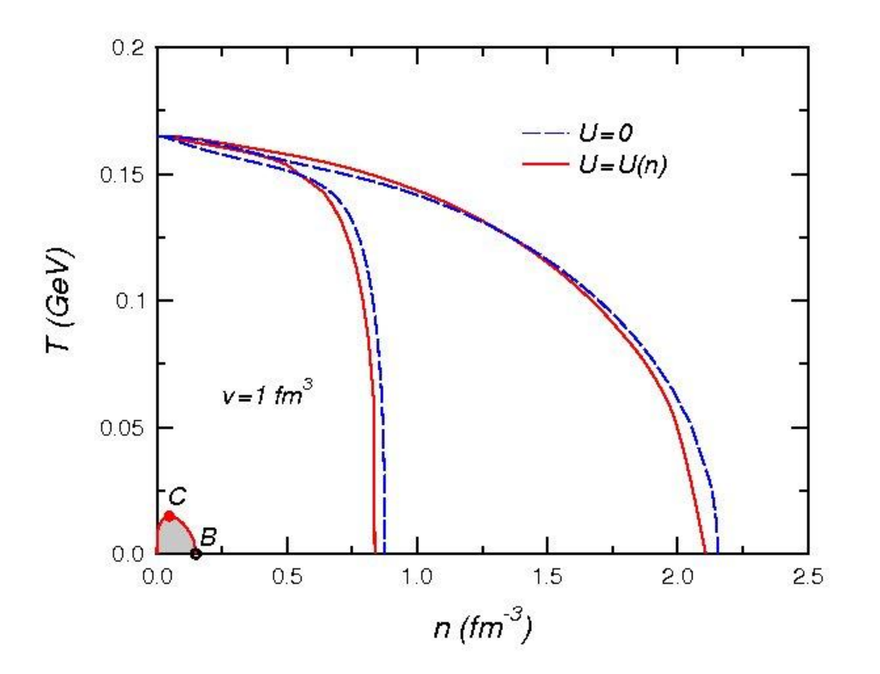
(similar to Barz, Schulz, Frieman, Knoll,1983)

This is different compared to chiral models like LσM or NJL

Scavenius, Mocsy, Mishustin, Rischke 2001

Phase diagram including mean-field effects



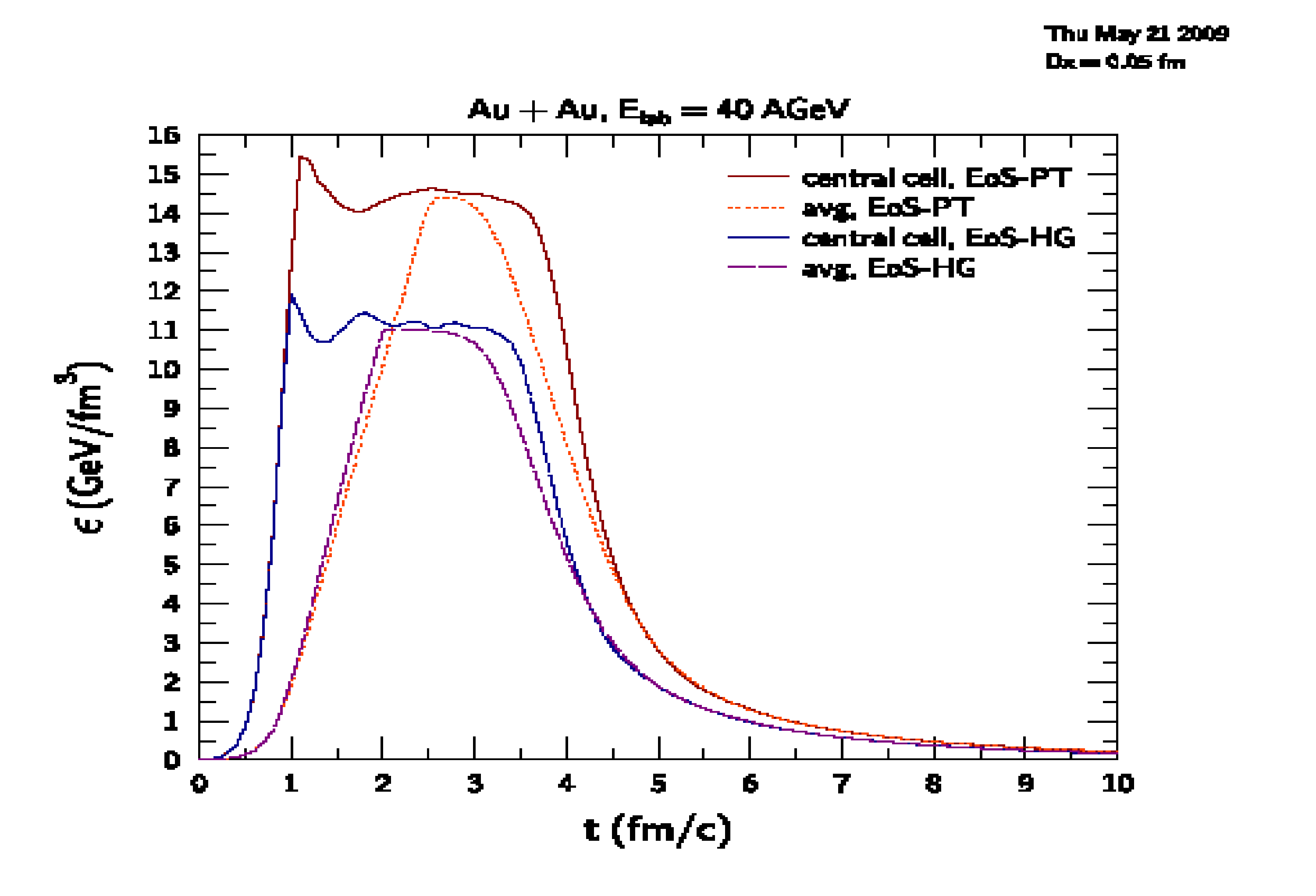


➡ Two phase transitions:

1) Deconfinement-hadronization phase transition at high μ and/or T

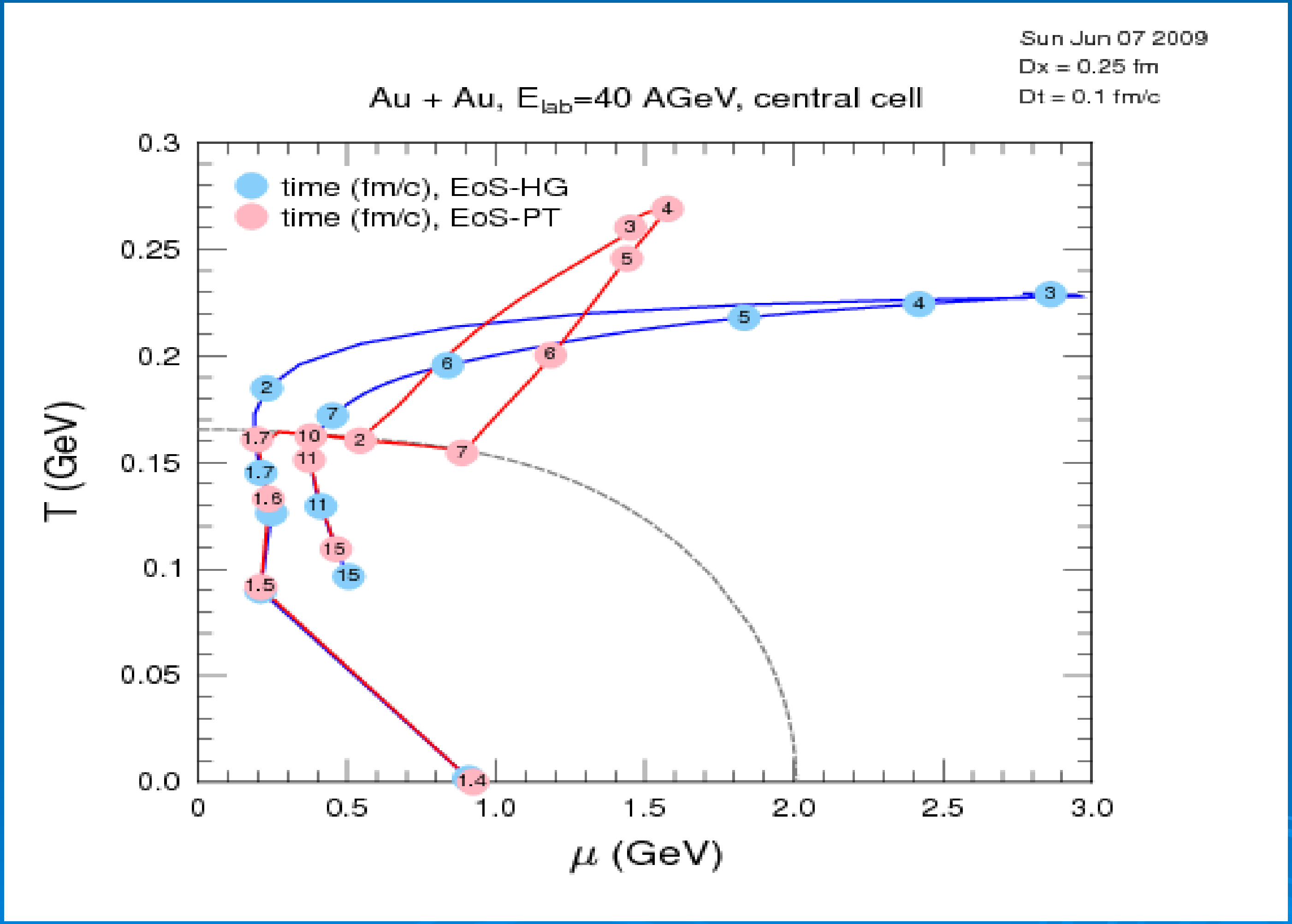
2) Liquid-gas phase transition at low T and μ≈m corresponding to nuclear (quarkyonic) matter

Energy density in central cell



➡ Energy densities above 2 GeV/fm exist only in the time interval between1 and 5 fm/c. Much higher ε are reached in the case of PT.

Dynamical trajectories of matter in central cell



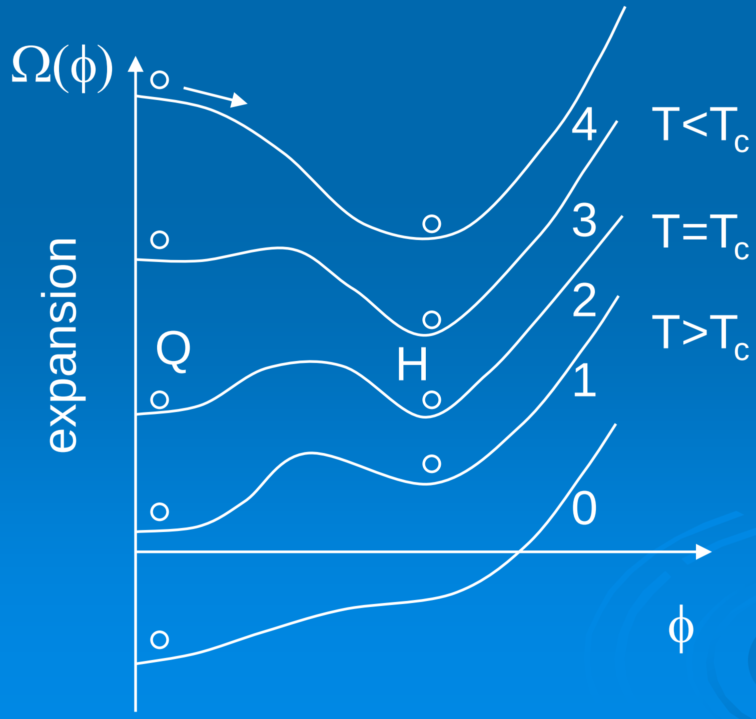
➡ In the equilibrium scenario the final state is not sensitive to the phase transition. Non-equilibrium effects may help!

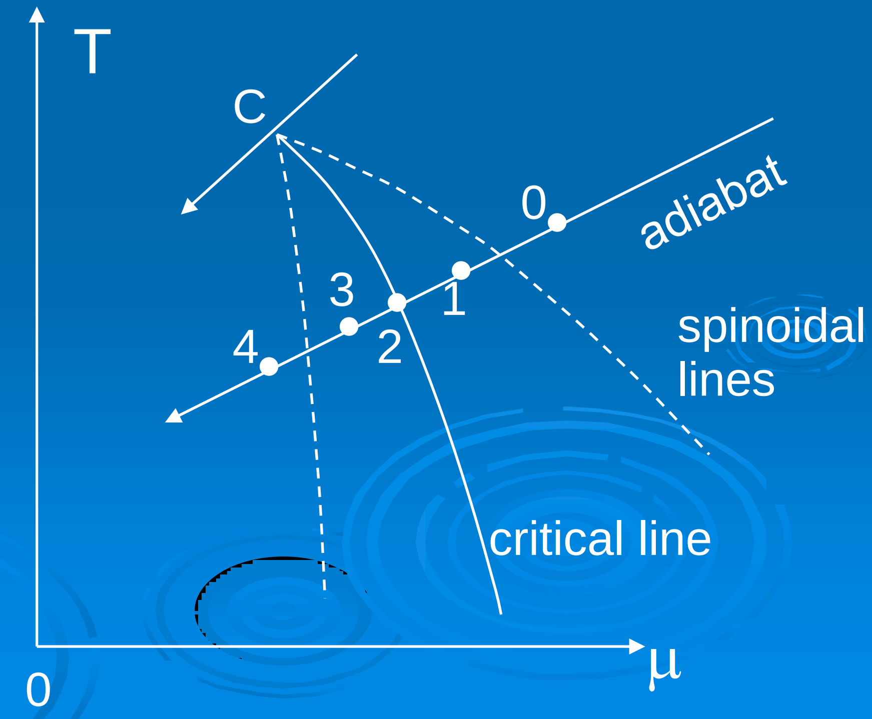
Spinodal decomposition

Effective thermodynamic potential leading to 1st order transition

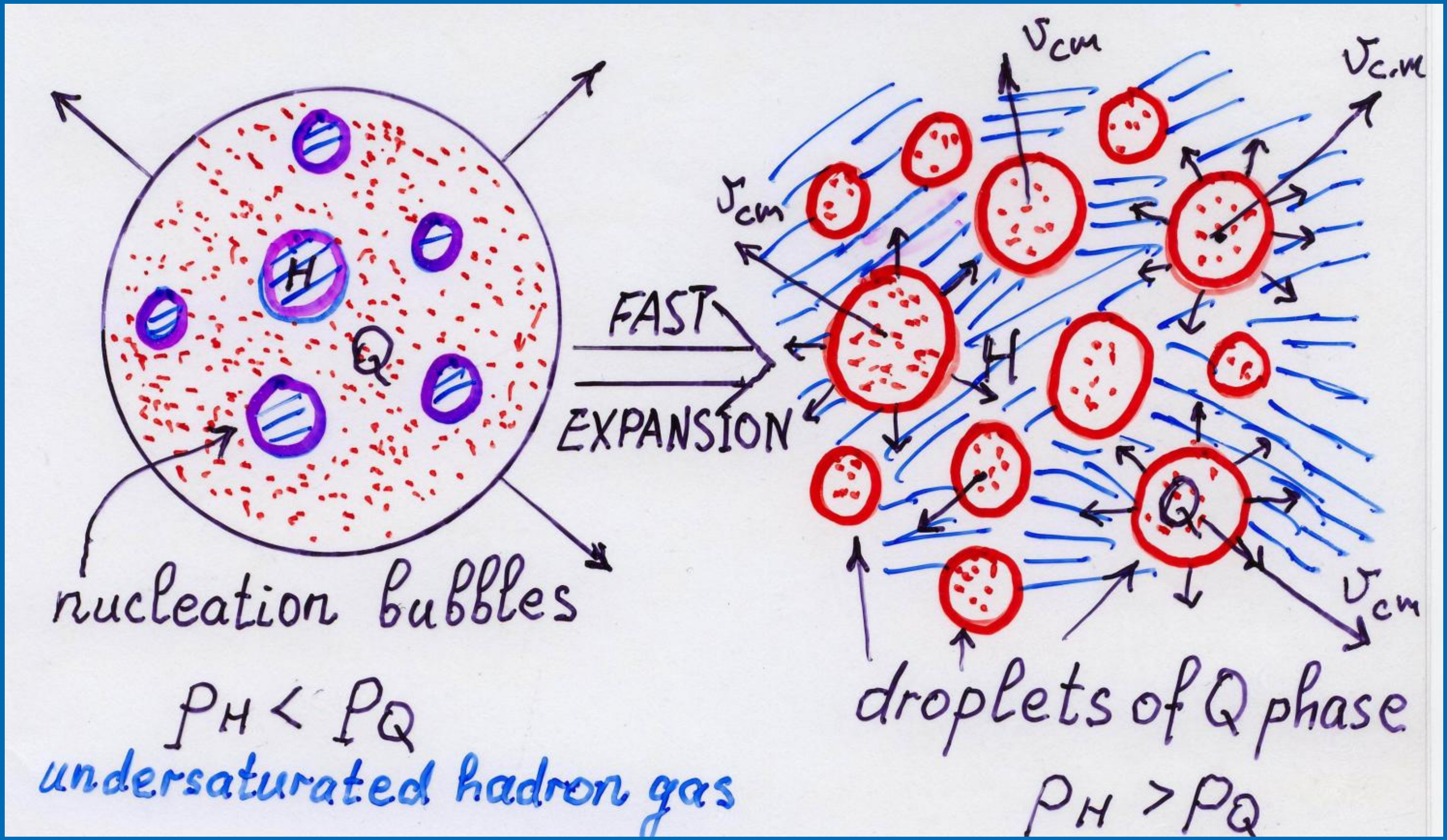
$$\Omega(\phi;T,\mu)=\Omega_0(T,\mu)+\frac{a}{2}\phi^2+\frac{b}{4}\phi^4+\frac{c}{6}\phi^6,$$

$$a,b,c-\text{functions of } T \text{ and } \mu$$





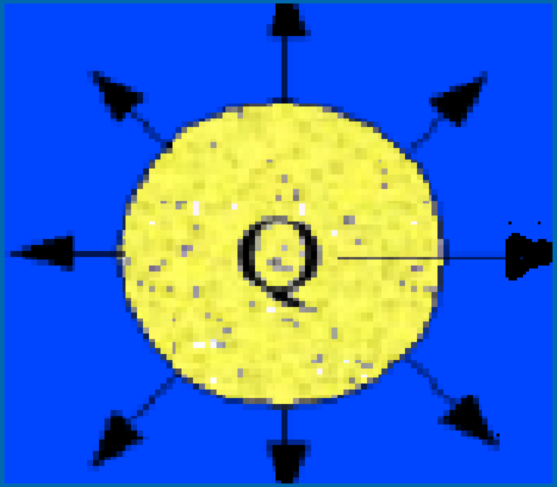
Dynamical fragmentation 1



In case of a 1st order phase transition the Q and H phases are separated by a barreer. In the course of fast expansion the transition is delayed until Q phase becomes unstable and then splits into quark droplets/hadrons

Csernai&Mishustin 1995, Mishustin 1999, Rafelski et al. 2000, Koch et al.2005

Dynamical fragmentation 2



Kinetic energy associated with collective expansion of an individual droplet is

$$E_{kin} = \frac{1}{2}\int_0^R \varepsilon_Q (Hr)^2 dV = \frac{2\pi}{5}\varepsilon_Q H^2 R^5$$

The surface energy is $E_{surf} = 4\pi R^2 \sigma$

Droplet size from the balance of kinetic and surface energies:

$$E_{kin} = E_{surf} \Rightarrow\Rightarrow R = \left(\frac{10\sigma}{\varepsilon_Q H^2}\right)^{1/3} \approx 1.5\ \mathrm{fm}$$

$$\sigma = 20\ \mathrm{MeV/fm}^2,\ \varepsilon_Q = 0.5\ \mathrm{GeV/fm}^3,\ H^{-1} \approx \tau_c \approx 3\ \mathrm{fm/c}$$

Bulk-viscosity driven clusterization 1

Lattice calculations suggest that bulk viscosity may strongly increase at Tc (Karsch, Kharzeev & Tuchin 2008)

$$\zeta(T) = \zeta_0 + \zeta_c \frac{T_C}{\sqrt{2\pi\sigma_T}} \exp\left[-\frac{(T-T_C)^2}{2\sigma_T{}^2}\right]$$

$$\zeta_0 \sim 10^{-3} s,\ \ \zeta_c \sim 0.3s,\ \ \sigma_T \approx 0.1T_c$$

At T=Tc the system suddenly becomes very stiff and cannot expand uniformly. It will break into pieces like a glass.

Bulk-viscosity driven clusterization 2

The fragment size is determined by the balance between the collective kinetic energy and the dissipated energy

(Torrieri, Tomazik&Mishustin 2008)

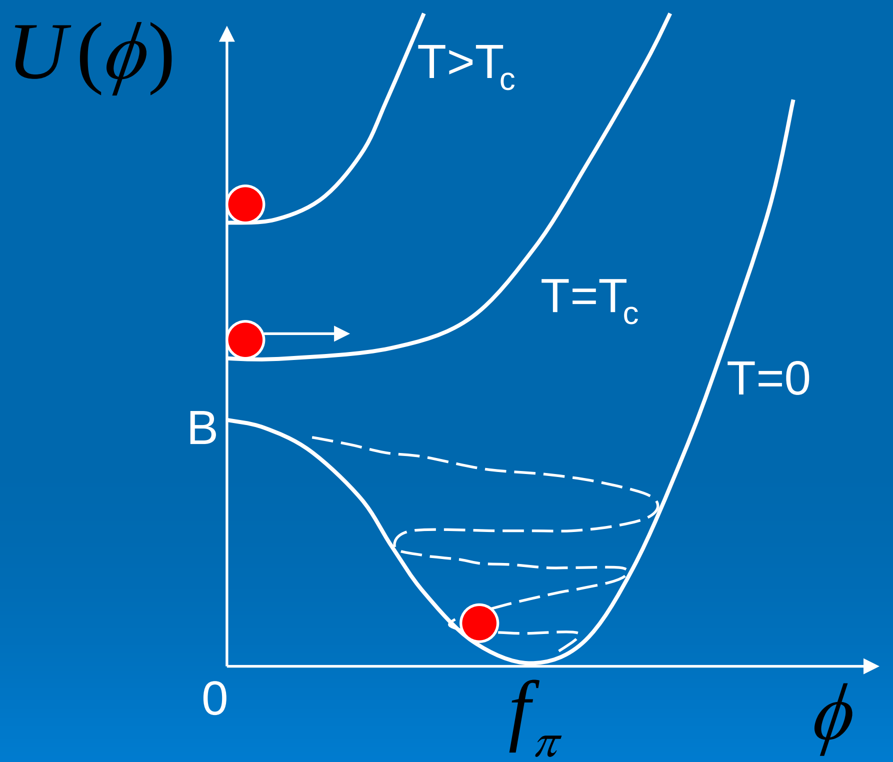
$$E_{dis} = \int dV \int d\tau \zeta (\partial_\mu u^\mu)^2 \approx V \frac{\zeta_c}{\tau_c} \left(\frac{d\ln\tau}{d\ln T}\right)_{T=T_C}$$

$$E_{kin} = \frac{1}{2}\int dV \varepsilon(\tau)(Hz)^2 \approx \frac{1}{3} V \varepsilon_c H^2 L^2$$

$$E_{dis} = E_{kin} \quad \rightarrow \quad L \approx \frac{3}{H}\sqrt{\frac{\zeta_c}{\tau_c \varepsilon_c}} \simeq (1\text{-}2)\ \text{fm}$$

$$\zeta_c \sim 0.3 s_c,\ T_c \approx 170\ \text{MeV},\ H^{-1} \approx \tau_c \approx 3\ \text{fm/c}$$

Critical slowing down in the 2nd order phase transition



Transition goes through the spinodal decomposition (Csernai&Mishustin 1995, Randrup 2003)

In the vicinity of the critical point the relaxation time for the order paprameter diverges

$$\tau_{\mathrm{rel}}(T) \sim \frac{1}{|T - T_c|^{\nu}}, \quad \nu \simeq 2$$

Strong fluctuations do not develop
Due to the "critical slowing down"
(Berdnikov, Rajagopal)

Fluctuations of the order parameter
Evolve according to the equation

$$\frac{d\delta\phi}{dt} = -\gamma \frac{\partial\Omega}{\partial\phi} \approx -\frac{\delta\phi}{\tau_{\mathrm{rel}}}$$

Non-statistical multiplicity fluctuations

- Assume that secondary hadrons are emitted from QGP droplets with masses following gamma-distribution

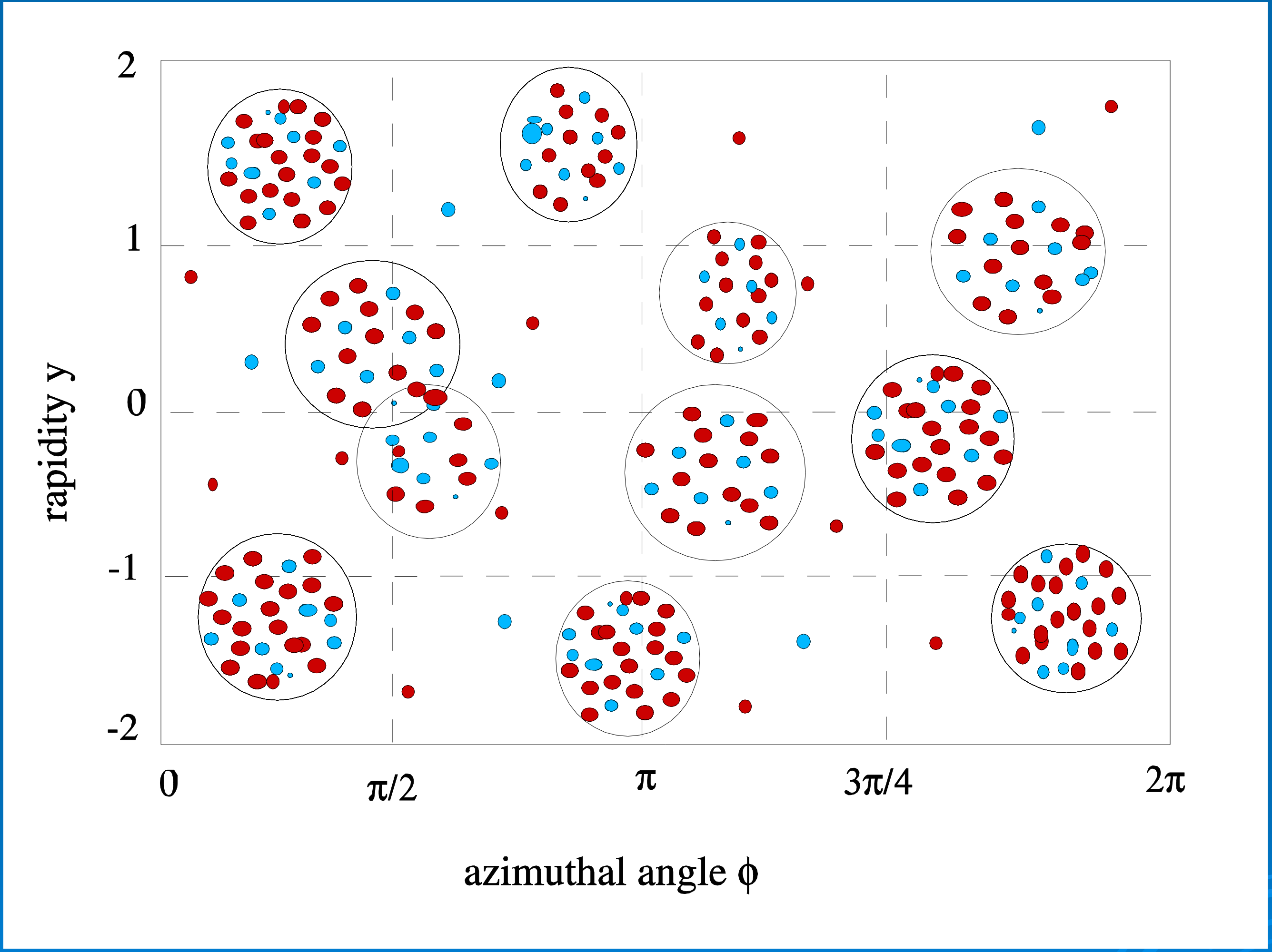
$$W_k(M) = \frac{b(bM)^{k-1}}{\Gamma(k)} \exp(-bM), \ \langle M \rangle = \frac{k}{b}, \ \sigma_{\mathrm{M}} = \frac{\langle M \rangle}{\sqrt{k}}$$

- Let the multiplicity of emitted hadrons from individual droplets follow Poissonian distribution $P_n(n)$ with $\bar{n} = \alpha M$
- Than the resulting multiplicity distribution is the Negative Binomial Distribution

$$Q_k(n) = \int_0^\infty dM W_k P_{\alpha M}(n) = \frac{(n+k-1)}{n!(k-1)!} \frac{(\bar{n}/k)^n}{(1+\bar{n}/k)^{n+k}},$$

$$\frac{\mathrm{var}(n)}{\langle n \rangle} = 1 + \frac{\langle n \rangle}{k} \ \text{and} \ \langle \mathrm{n} \rangle = \frac{\alpha k}{b}$$

Lego plot in eta-phi plane

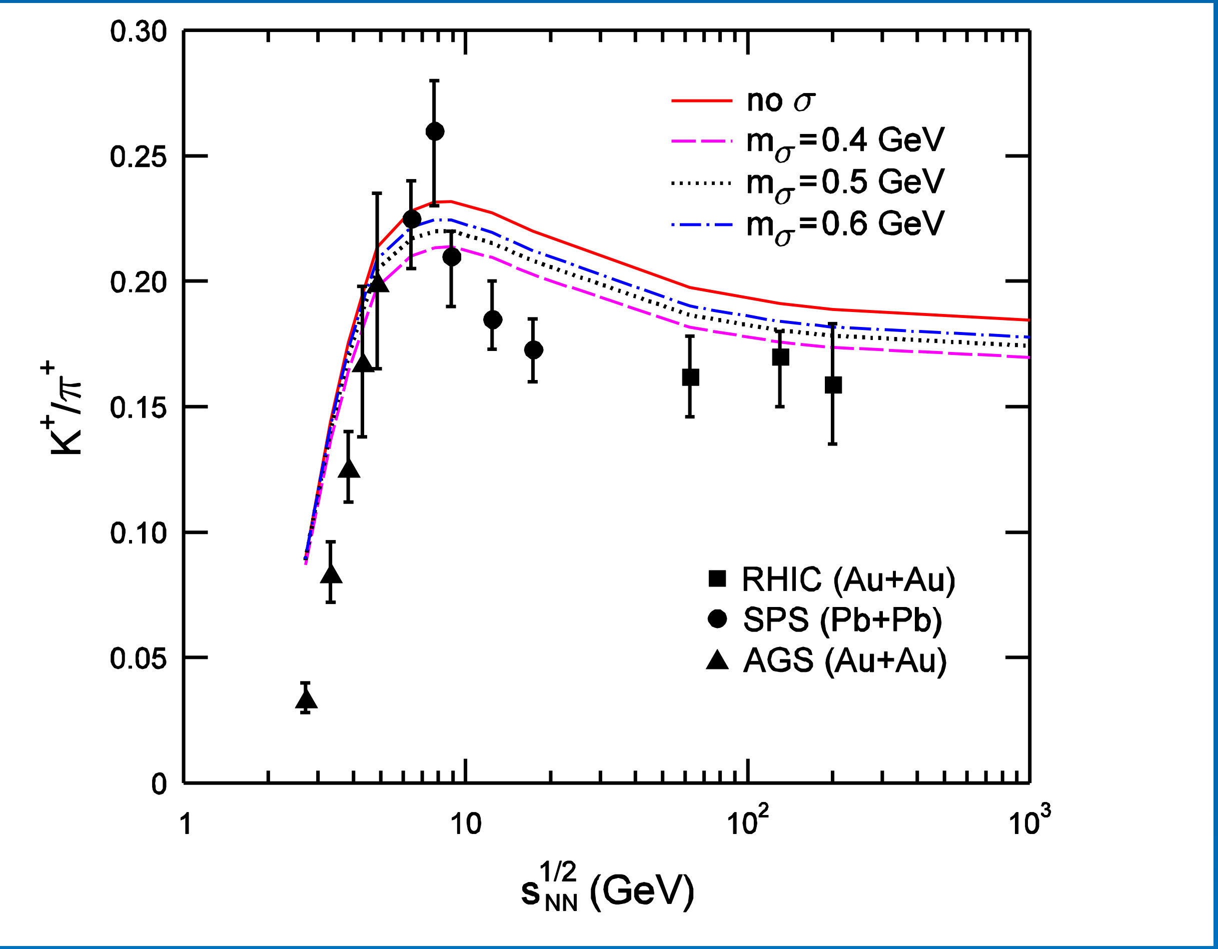


Event-by-event fluctuations of hadron distributions in momentum space associated with emission from quark droplets

Conclusions

- Hydrodynamical modeling is very useful tool for understanding complicated dynamics of HI collisions
- In equilibrium scenario manifestations of the phase transition are rather weak
- Non-equilibrium effects like clusterization of the QGP and its direct conversion into hadrons may help to identify this phase
- Strong non-statistical multiplicity fluctuations represent a very promising signal of the deconfinement transition
- Low energy program at RHIC and FAIR/CBM experiment will certainly help to find the deconfinement signals

K^+ / π^+ ratio and the "Horn"



- The data are reproduced reasonably well without any special effort, but the peak is not as sharp as data points
- The fit is improved by inclusion of $f_0(600)$, similar to Andronic, Braun-Munzinger, Stachel , arXiv0812.1186 [nucl-th]
- Better agreement at the left slop can be achieved by introducing $\gamma_S < 1$